

**Exercise 1** *DUNE grid interface (practical exercise)*

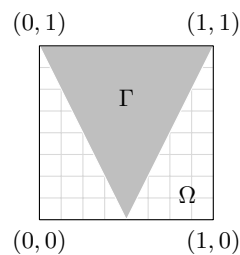
In this exercise you should become familiar DUNE grid interface. For our practical exercise we will use DUNE module *dune-mpde* (see homepage).

In the directory */dune-mpde/uebungen/uebung01* you will find an example programm which integrates the analytical function

$$f(x, y) = \exp^{-3.234((x-0.5)^2+(y-0.5)*2)}$$

on a structured grid. The integral is approximated by first order quadrature. The integration domain is unit cube  $\Omega = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 1\}$ . The integration is done for several grid refinements (beginning with only one cell) to prove the convergence. In addition, the function  $f_h$  (approximation of  $f$  in grid vertices) will be stored as VTK file for visualization purposes.

1. Have a look at the programm, try to understand the source code, compile it and run the programm. Start *paraview* to view the VTK file. Try to use different filters (especially *warp* filter).
2. Modify the program in the way that the function  $f$  will be integrated over the triangle subdomain  $\Gamma \subset \Omega$  (see picture below).



Implement following integration domains:

- i) Only cells that are completely in  $\Gamma$  will be considered to quadrature.
  - ii) Only cells with at least one vertex in  $\Gamma$  will be considered to quadrature.
3. Instead of a structured grid, you should integrate over an unstructured triangle grid which covers  $\Gamma$  completely. The file *triangle.msh* contains the suitable grid. This file can be read with the following commands:

```
typedef Dune::UGGrid<2> GridType;
GridType grid;
// read gmsh file
Dune::GridFactory<GridType> factory(&grid);
Dune::GmshReader<GridType>::read(factory, "triangle.msh", true, true);
factory.createGrid();
```

Compare the convergence orders of structured and unstructured grid. Try different integration orders. Does it make any difference? Why?

4. Modify the class *FunctorVTKFunction* such that the output will be zero for all cells which are not completely in  $\Gamma$ .

( 20 Points )