

## Exercise Sheet 2

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### Exercise 1: C++ Quiz

(5 points)

On <https://cppquiz.org> you can find a quiz with C++ specific questions. In this exercise, answer the following questions:

**Question 1:** <https://cppquiz.org/quiz/question/197> (variable lifetime)

**Question 2:** <https://cppquiz.org/quiz/question/161> (Duff's Device)

**Question 3:** <https://cppquiz.org/quiz/question/9> (reference arguments)

**Question 4:** <https://cppquiz.org/quiz/question/113> (overload resolution)

**Question 5:** <https://cppquiz.org/quiz/question/5> (initialization order)

The questions are sorted (more or less) according to the structure of the lecture. For questions 1, 3, 4, and 5, write a short statement what information the question and its solution are trying to convey. Regarding question 2: inform yourself about the construct that is used. What is its purpose? Would you suggest using this in real-world code? Why, or why not?

### Exercise 2: Rational Numbers

(10 points)

Write a class for rational numbers. The number should always be represented as a *fully reduced fraction* of the form

$$\frac{\text{numerator}}{\text{denominator}}$$

with denominator  $> 0$ .

- (a) What is an appropriate data structure for rational numbers?
- (b) Start by writing a function `int gcd(int, int)` (greatest common divisor), you will need it to reduce fractions.
  - You can use the Euclidean algorithm to determine the greatest common divisor.
  - For an algorithm see [https://en.wikipedia.org/wiki/Greatest\\_common\\_divisor](https://en.wikipedia.org/wiki/Greatest_common_divisor)
  - Implement this scheme as a recursive function.
- (c) Write a class `Rational`, which represents a rational number. The constructor should have the numerator and the denominator as arguments. Be sure to check for valid input. In addition, the class has two functions `numerator()` and `denominator()` that return the values of the numerator and denominator. The class should have three constructors:
  - a default constructor that initializes the fraction with 1,
  - a constructor that initializes the fraction with a given numerator and denominator, and
  - a constructor that initializes the fraction with a given whole number.
- (d) Supplement the class with operators for `*=` `+=` `-=` `/=` and `==`.
- (e) Use the newly implemented methods to implement free operators `*` `+` `-` `/`.

(f) Check your implementation using various test cases. Initialize three fractions

$$f_1 = -\frac{3}{12}, \quad f_2 = \frac{4}{3}, \quad f_3 = \frac{0}{1}.$$

Test the operators with the following examples:

$$f_3 = f_1 + f_2, \quad f_3 = f_1 \cdot f_2, \quad f_3 = 4 + f_2, \quad f_3 = f_2 + 5, \quad f_3 = 12 \cdot f_1, \quad f_3 = f_1 \cdot 6, \quad f_3 = \frac{f_1}{f_2}.$$

Print the result after each operation. The corresponding solutions are:

$$\frac{13}{12}, \quad -\frac{1}{3}, \quad \frac{16}{3}, \quad \frac{19}{3}, \quad -\frac{3}{1}, \quad -\frac{3}{2}, \quad -\frac{3}{16}.$$

### Exercise 3: Farey Sequences

(10 points)

A Farey sequence  $F_N$  of degree  $N$  (or: the Farey fractions of degree  $N$ ) is an ordered set of reduced fractions

$$\frac{p_i}{q_i} \quad \text{with} \quad p_i \leq q_i \leq N \quad \text{and} \quad 0 \leq i < |F_N|$$

and

$$\frac{p_i}{q_i} < \frac{p_j}{q_j} \quad \forall 0 \leq i < j < |F_N|.$$

Use the class `Rational` from the previous exercise to write a function

```
void Farey(int N)
```

which calculates the Farey fractions up to degree  $N$  and prints the resulting Farey sequences up to degree  $N$  on the screen.

*Algorithm:* The sequences can be computed recursively. The first sequence is given by

$$F_1 = \left( \frac{0}{1}, \frac{1}{1} \right)$$

For a known sequence  $F_N$  one can get  $F_{N+1}$  by inserting an additional fraction  $\frac{p_i+p_{i+1}}{q_i+q_{i+1}}$  between two consecutive entries  $\frac{p_i}{q_i}$  and  $\frac{p_{i+1}}{q_{i+1}}$  if  $q_i + q_{i+1} = N + 1$  holds for the sum of denominators.

*Example:* Determining  $F_7$  from  $F_6$  results in the following construction:

$$F_6 = \left( \underbrace{\frac{0}{1}, \frac{1}{6}}_{\frac{1}{7}}, \underbrace{\frac{1}{4}, \frac{1}{3}}_{\frac{2}{7}}, \underbrace{\frac{2}{5}, \frac{1}{2}, \frac{3}{5}}_{\frac{3}{7} \text{ and } \frac{4}{7}}, \underbrace{\frac{2}{3}, \frac{3}{4}}_{\frac{5}{7}}, \underbrace{\frac{4}{5}, \frac{5}{6}, \frac{1}{1}}_{\frac{6}{7}} \right)$$

The new elements are:

$$\frac{0+1}{1+6} = \frac{1}{7} ; \quad \frac{1+1}{4+3} = \frac{2}{7} ; \quad \frac{2+1}{5+2} = \frac{3}{7} ; \quad \frac{1+3}{2+5} = \frac{4}{7} ; \quad \frac{2+3}{3+4} = \frac{5}{7} ; \quad \frac{5+1}{6+1} = \frac{6}{7}$$

The sorted sequence then is:

$$F_7 = \left( \frac{0}{1}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{2}{7}, \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{1}{1} \right)$$

*For checking:*

The Farey sequences up to degree 6

$$F_1 = \left( \frac{0}{1}, \frac{1}{1} \right)$$

$$F_2 = \left( \frac{0}{1}, \frac{1}{2}, \frac{1}{1} \right)$$

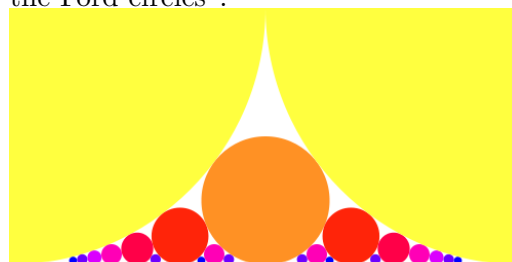
$$F_3 = \left( \frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1} \right)$$

$$F_4 = \left( \frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1} \right)$$

$$F_5 = \left( \frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1} \right)$$

$$F_6 = \left( \frac{0}{1}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{1}{1} \right).$$

There is a beautiful illustration of these fractions, the Ford circles<sup>a</sup>:



<sup>a</sup>see [https://en.wikipedia.org/wiki/Ford\\_circle](https://en.wikipedia.org/wiki/Ford_circle)