Prof. Dr. Peter Bastian, Linus Seelinger
IWR, Universität Heidelberg

Submission date: 12 July 2018

Note: Do not forget to update your dune installation as described in exercise sheet 2.

Exercise 1 *Transformation between Lagrange and Hierarchical Basis*

Let a 1D coarse grid with N elements of width H be given. The finer grids of width $\frac{H}{2^l}$ are generated through uniform refinement. On these grids it is possible to use both the standard basis and the hierarchical basis. See figure 1 for a representation of the bases based on a coarse grid consisting of 2 elements.

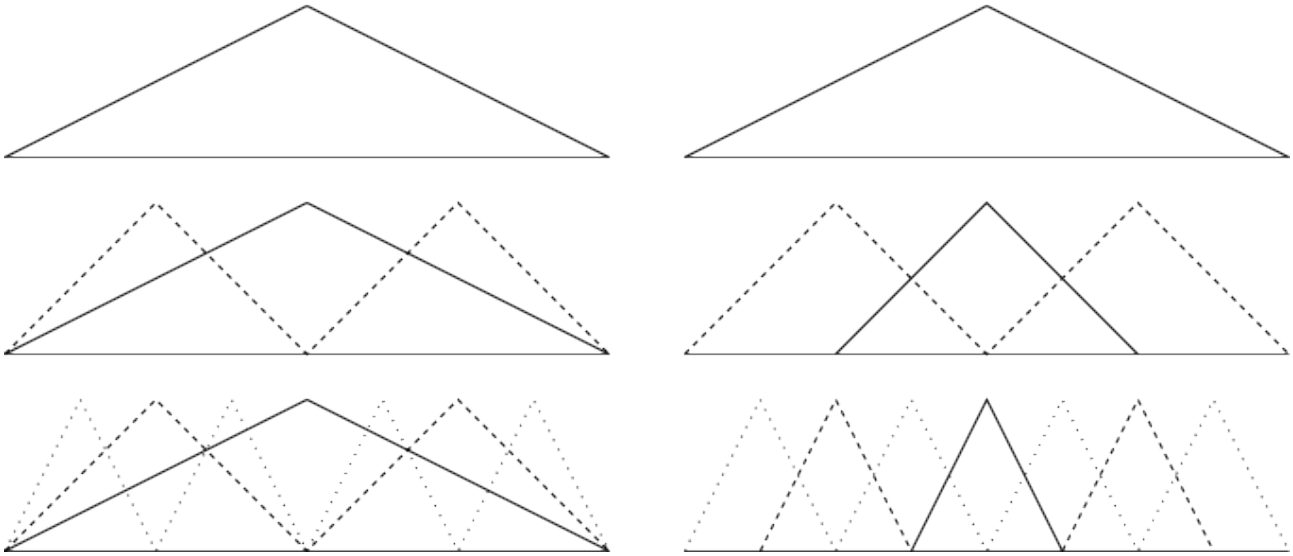

Figure 1: Hierarchical basis (left) versus standard nodal basis (right) in 1D

Calculate the transformation between these two bases on the grid level l. **(7 Points)**

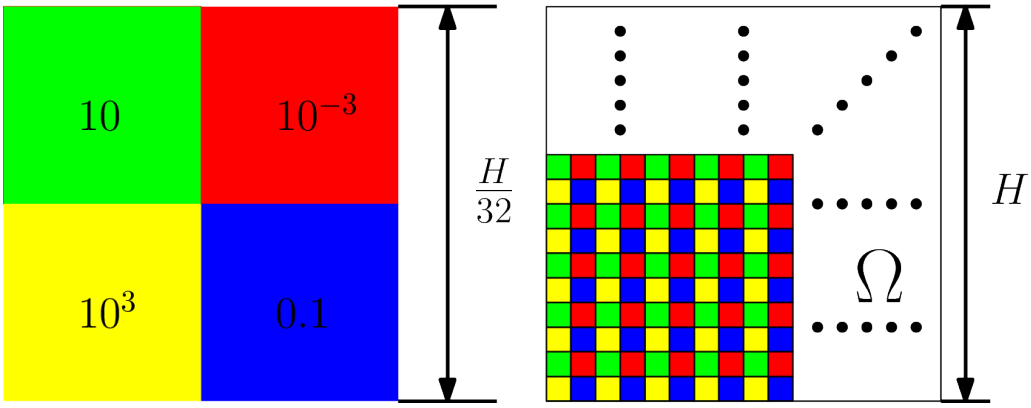

Figure 2: Permeability field in the domain Ω (cube with side length H).

Exercise 2 *Solver robustness for diffusion problems with heterogeneous permeability field*

The code for this week's exercise can be found in the directory `uebungen/uebung06`. It provides working implementations of four different parallel solvers, namely

- the additive Schwarz method,
- the additive Schwarz method with coarse grid correction,
- the Multilevel Diagonal Scaling (MDS) method,
- the multiplicative multigrid method.

In this exercise we want to solve the elliptic problem

$$\begin{aligned} -\nabla \cdot (A(x)\nabla u(x)) &= 0 && \text{in } \Omega = (0,1)^d, \\ u(x) &= \exp(-\|x\|_2^2) && \text{on } \partial\Omega. \end{aligned}$$

The parameters for this problem are provided in the class *GenericEllipticProblem* in the header file `problem1.hh` where $A = I_d$ as in the previous exercises. Purpose of this exercise is to investigate the robustness of the solvers under anisotropies coming from a space-dependent diffusion tensor.

Task 1 Modify the problem such that the permeability field A is heterogeneous. The space-dependent scalar $\lambda(x)$ in the diffusion tensor should represent the *checkerboard pattern*, thus it can take the four values $\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}$ in general. These values can be changed with the configuration files `additive_schwarz.ini` and `multilevel_settings.ini`.

Implement the checkerboard pattern with arbitrary values $\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}$ as presented in figure 2. **Note** that figure 2 shows the case $\lambda_{11} = 10, \lambda_{12} = 10^{-3}, \lambda_{21} = 10^3, \lambda_{22} = 0.1$.

Task 2 Present the number of iterations for each solver for various realizations of the checkerboard pattern in form of a table. Suggestions:

- $\lambda_{11} = \lambda_{21}, \lambda_{12} = \lambda_{22}$ and $\lambda_{11} = \lambda_{22}, \lambda_{12} = \lambda_{21}$
- the realization presented in figure 2

Investigate the solvers' behaviour with respect to changing parameter contrast ($\frac{\lambda_{11}}{\lambda_{12}}$), overlap and number of subdomains.

(15 Points)