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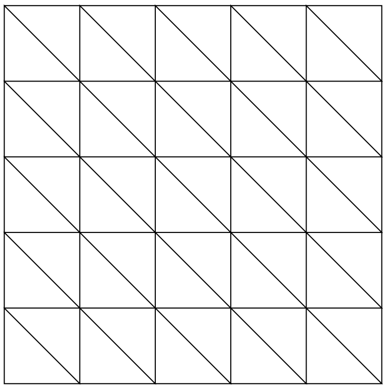
Exercise 1 *P_1 Finite Elements on a structured simplicial mesh*

Consider the Poisson equation

$$
\begin{aligned}
-\Delta u &= f && \text{in } \Omega = (0,1)^2 \subset \mathbb{R}^2, \\
u &= 0 && \text{auf } \partial\Omega\ .
\end{aligned}
$$

We want to solve this equation numerically with P_1 Finite Elements. The unitsquare Ω is discretized with the following *structured triangular mesh*:



Let N be the number of divisions in x- and y-direction (in the picture we have $N = 5$), thus the mesh size is $h = \frac{1}{N}$. We number the nodes starting from the origin row-wise beginning at 0 to $(N+1)^2 - 1$. Your task is to specify one row of the stiffness matrix belonging to an interior node of the mesh.

(10 Points)

Exercise 2 *Computation of the L^2-norm in DUNE*

First of all, get familiar with the structure of the program `uebung01/uebung01.cc`. What PDE does it solve (see `problem.hh`)? What reference solution is used for the error calculation?

Complete the implementation of `domain_volume`, while revisiting how to access a grid and the elements' geometries.

Finish the implementation of the calculation of the L^2 error norm. This makes use of the `GridFunction` concept where functions are defined on a per-element basis. Why is this a reasonable interface? Why not just go for a global evaluation as in the `AnalyticGridFunction`

Plot L^2 error vs. mesh size. Does the result match your expectations? What happens for higher polynomial degree?

Bonus task: Implement a template function `l2norm()` that computes the L^2-norm of a globally defined function directly. You can start from a copy of `domain_volume` and extend it to additionally receive a function like the already defined `ExactSolution`. The integral can be approximated by quadrature over the center points of elements, which you can retrieve from the `Geometry` class.

(10 Points)