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Exercise 1 *Additive Schwarz iteration*

As defined in the lecture, additive Schwarz iteration reads

$$x^{k+1} = x^k + \omega \sum_{i=1}^{p} R_{\hat{I}_i}^T A_{\hat{I}_i \hat{I}_i}^{-1} R_{\hat{I}_i} (b - Ax^k),$$

with $\omega \in \mathbb{R}^+$ being the damping factor. Using that A is s.p.d., prove that the matrix

$$B = \sum_{i=1}^{p} R_{\hat{I}_i}^T A_{\hat{I}_i \hat{I}_i}^{-1} R_{\hat{I}_i}$$

is symmetric positive definite. **(4 Points)**

Exercise 2 *Multiplicative Schwarz as Gauss-Seidel iteration*

In the lecture you have learned the multiplicative Schwarz method applied to an overlapping decomposition of the domain Ω with p subdomains (c.f. Algorithm 3.2 in the script).

In this exercise we are going to combine the multiplicative Schwarz method together with one dimensional subspace correction methods.

Let $V_h = \mathrm{span}\{\varphi_i^h, i \in I_h\}$ be the Finite Element space of the triangulation $\mathcal{T}_h$ of the domain Ω. For the subspaces, set $V_i = \mathrm{span}\{\varphi_i^h\}$ for $i \in I_h$. This decomposition has the property

$$V_h = \bigoplus_{i=1}^{N_h} V_i,$$

thus the index sets $I_i = \{i\}$ belonging to the V_i are now non-overlapping and the decomposition

$$u_h = \sum_{i=1}^{N_h} u_i \quad , \; u_i \in V_i$$

is unique.

1. Show that one iteration of the multiplicative Schwarz method in this case is equivalent to the Gauss-Seidel iteration

$$x^{(k+1)} = x^{(k)} + W^{-1}(b - Ax^{(k)})$$

   where $W = D + L$.

2. Generalize the previously shown multiplicative formula of the Gauss-Seidel iteration to show that the multiplicative Schwarz method for an overlapping decomposition with p subdomains is equivalent to one block Gauss-Seidel step with overlapping index sets.

(7 Points)