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Submission date: 4 July 2019

Exercise 1 *Transformation between Lagrange and Hierarchical Basis*

Let a 1D coarse grid with N elements of width H be given. The finer grids of width $\frac{H}{2^l}$ are generated through uniform refinement. On these grids it is possible to use both the standard basis and the hierarchical basis. See figure 1 for a representation of the bases based on a coarse grid consisting of 2 elements.

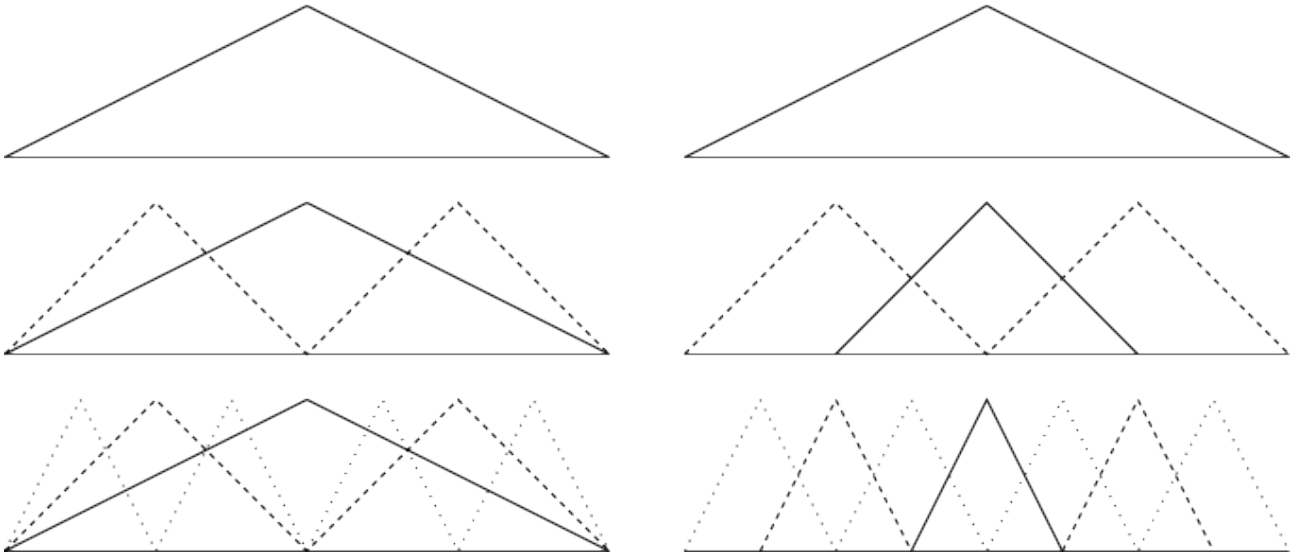

Figure 1: Hierarchical basis (left) versus standard nodal basis (right) in 1D

Calculate the transformation between these two bases on the grid level l. **(7 Points)**

Exercise 2 *Parallel Multigrid*

In the lecture the restrictions $r_{l,i}$, $R_{l,i}$ and R_l have been introduced. With $r_{l,i} : \mathbb{R}^{I_l} \to \mathbb{R}^{I_{l,i}}$ we denote the restriction to the subdomain i, such that for $x_l \in \mathbb{R}^{I_l}$ it holds

$$(r_{l,i}x_l)_j = (x_l)_j \quad \forall j \in I_{l,i}$$

as in the Schwarz methods. The multilevel restriction $R_l : \mathbb{R}^{I_{l+1}} \to \mathbb{R}^{I_l}$ is defined as

$$(R_l x_{l+1})_\alpha = \sum_{\beta \in I_{l+1}} \theta^{l,l+1}_{\alpha,\beta} (x_{l+1})_\beta$$

for $x_{l+1} \in \mathbb{R}^{I_{l+1}}$. The restriction of R_l to the subdomain i is denoted by $R_{l,i} : \mathbb{R}^{I_{l+1,i}} \to \mathbb{R}^{I_{l,i}}$ and is defined for $x_{l+1,i} \in \mathbb{R}^{I_{l+1,i}}$ as

$$(R_{l,i}x_{l+1,i})_\alpha = \sum_{\beta \in I_{l+1,i}} \theta^{l,l+1}_{\alpha,\beta} (x_{l+1,i})_\beta.$$

In this exercise we consider additional properties of these operators besides Observation 6.4 and Observation 6.5 in the lecture notes.

1. Show that the following equality does **not** hold in general,

$$R_{l,i}\, r_{l+1,i}\, x_{l+1} = r_{l,i}\, R_l\, x_{l+1}. \tag{1}$$

   Hint: It is sufficient to consider this in one dimension.

2. Let $\hat{I}_{l,i} \subset I_{l,i}$ have the properties: $\alpha \in \hat{I}_{l,i} \Rightarrow s_a \in \Omega_i \wedge s_a \notin \partial\Omega_i$. Then (1) holds $\forall \alpha \in \hat{I}_{l,i}$.

3. Describe the consequences implied by these properties for the implementation of **overlapping** multigrid methods.

(8 Points)