Prof. Dr. Peter Bastian, Linus Seelinger Submission date: 18 July 2019

IWR, Universität Heidelberg

Exercise 1 *Schwarz Method with Inexact Local Solvers*

Let $a(\cdot,\cdot)$ a bilinear form and A the associated stiffness matrix. In the Schwarz method with exact solutions on subdomains, a local bilinear form $a_i(\cdot,\cdot)$ mit

$$a_i(u_i, v_i) := a(R_i^T u_i, R_i^T v_i)$$

and its associated stiffness matrix

$$A_i = R_i A R_i^T$$

are considered. Now assume that an inexact solver instead of $a_i(\cdot,\cdot)$ solves a perturbed $\tilde{a}_i(\cdot,\cdot)$ with stiffness matrix $\tilde{A}_i$. We still assume $\tilde{a}_i$ to be symmetric positive definite.

What are the operators P_i of the Schwarz method in this case? What properties of the P_i remain, which can be lost due to inexact solving? What parts of the convergence proof for Schwarz methods are affected?

(8 Points)

Exercise 2 *AMG Coarsening*

We consider the usual 2D Laplace problem on a unit square discretized by Finite Differences using a uniform square grid.

The resulting matrix exhibits the Finite Difference stencil

$$\frac{1}{h^2}\begin{bmatrix} & -1 & \\ -1 & 4 & -1 \\ & -1 & \end{bmatrix}.$$

For a $5x5$ vertices example, construct a partitioning of all degrees of freedom into a coarse and fine index set $V = F \cup C$ according to the criterion given in the script. Choose an appropriate α for the algorithm to work reasonably.

Show your results as a graphical sketch.

(5 Points)