Parallel Solvers for Finite Elements (WS 2020)   **Exercise 1**

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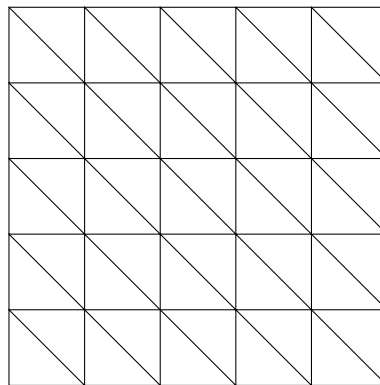IWR, Universität Heidelberg

Exercise 1   *P_1 Finite Elements on a structured simplicial mesh*

Consider the Poisson equation

$$-\Delta u = f \qquad \qquad \text{in } \Omega = (0,1)^2 \subset \mathbb{R}^2,$$
$$u = 0 \qquad \qquad \text{auf } \partial\Omega .$$

We want to solve this equation numerically with P_1 Finite Elements. The unitsquare Ω is discretized with the following *structured triangular mesh*:

Let N be the number of divisions in x- and y-direction (in the picture we have $N = 5$), thus the mesh size is $h = \frac{1}{N}$. We number the nodes starting from the origin row-wise beginning at 0 to $(N+1)^2 - 1$. Your task is to specify one row of the stiffness matrix belonging to an interior node of the mesh.

(10 Points)

Exercise 2   *Computation of the L^2-norm in DUNE*

First of all, get familiar with the structure of the program `uebung01/uebung01.cc`. What PDE does it solve (see `problem.hh`)? What reference solution is used for the error calculation?

Complete the implementation of `domain_volume`. For this you might need to look into the class documentation of Dune [†].

Finish thee implementation of the calculation of the L^2 error norm. This makes use of the `GridFunction` concept where functions are defined on a per-element basis. Why is this a reasonable interface? Why not just go for a global evaluation as in the `AnalyticGridFunction`

Plot L^2 error vs. mesh size. Does the result match your expectations? What happens for higher polynomial degree?

Bonus task: Implement a template function `l2norm()` that computes the L^2-norm of a globally defined function directly. You can start from a copy of `domain_volume` and extend it to additionally receive a function like the already defined `ExactSolution`. The integral can be approximated by quadrature over the center points of elements, which you can retrieve from the `Geometry` class.

(10 Points)

[†] `https://www.dune-project.org/doxygen/pdelab/master/classes.html`