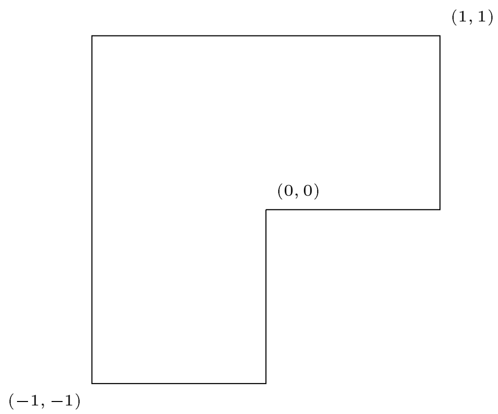
EXERCISE 1 A-POSTERIORI ERROR ESTIMATION FOR LAPLACE EQUATION

In the lecture *a posteriori* error estimator for the second order elliptic boundary problem was derived. In this exercise we consider the Laplace equation with Dirichlet boundary conditions:

$$-\Delta u = 0, \quad x \in \Omega$$
$$u(x) = g(x), \quad x \in \partial\Omega$$

on a polygon domain $\Omega \subset \mathbb{R}^d$ (not necessary convex). We restrict our estimation to P^1 finite elements. Follow the lecture and derive the *a posteriori* error estimation for the error $e_h = u - u_h$. *4 points*

EXERCISE 2 RESIDUAL ESTIMATION



In this exercise, we will solve reentrant corner problem using grid adaptation. We consider the Poissson problem in two space dimensions $\Delta u = 0$ on domain Ω (see picture) with the exact solution in polar coordinates

$$g(r, \phi) = r^{\frac{2}{3}} \sin(\frac{2}{3}\phi).$$

In *uebungen/uebung11* of your *dune-npde* module you can find a program that solves this problem for conform P_1 elements on the simplex mesh. Your task is to implement the *a-posteriori* error estimation for P_1 elements and use it to the local mesh adaptation.

1. The code is almost complete. You should only complete the implementation of the function `computeLocalError()` in the file *local_error.hh*, that should compute the residual error indicators η_t from the lecture.
2. The function `adaptGrid()` adapts the mesh dependent on values in vector `indicators`. You can decide between two strategies which cells to refine. Describe the difference between these two strategies.
3. Choose one of the strategies and compare the achieved accuracy (with respect to the number of degrees of freedom) to results using global mesh refinement.

6 points