EXERCISE 1 INTERPOLATION ON TRIANGLE

Let $v \in C^2(K)$ and K be a triangle with vertices $a_1, a_2, a_3 \in \mathbb{R}^2$. Functions ϕ_i for $i = 1, 2, 3$ denote $P^1(K)$ basis functions satisfying $\phi_i(a_j) = \delta_{ij}$. The longest side of triangle K is h_K and the smallest angle is τ_K. The P^1-interpolation function has a form

$$\Pi v(x) = \sum_{i=0}^{3} v(a_i)\phi_i(x).$$

Prove the following estimations:

1. $$\|v - \Pi v\|_{L_\infty(K)} \leqslant \frac{1}{2} h_K^2 \|D^2 v\|_{L_\infty(K)}$$

2. $$\|\nabla(v - \Pi v)\|_{L_\infty(K)} \leqslant \frac{3}{\sin \tau_K} h_K \|D^2 v\|_{L_\infty(K)}$$

5 points

EXERCISE 2 LOSS OF COERCIVITY, CROSS-POINT PROBLEM

Notice: although this is a practical exercise, you do not need to programm anything by yourself!

In *uebungen/uebung09* of your *dune-npde* module you can find a program that solves so-called cross point problem (for details see lecture notes Example 8.20) with P^k finite element on a conform trianglular grid (*UGGrid*) and with Q^k finite element on a conform quadrilateral gride (*YaspGrid*) and computes $\|u - u_h\|_{0,\Omega}$, $\|\nabla(u - u_h)\|_{0,\Omega}$, $\|u - u_h\|_{1,\Omega}$ and $\|u - u_h\|_{L_\infty(\Omega)}$ norms and its rates.

You can change input parameters in file *uebung09.ini*.

1. Compile the program and run it. Where in the domain is the singularity (look at $u - u_h$ in paraview)?

2. In homogeneous case ($k_1 = 1, k_2 = 1$), the convergence rates are kind of strange. Can you explain it?

3. Run the programm for different polynomial degrees 1 and 2 with different permeabilities ($k_1 = 1, k_2 = \{1, 100, 10000\}$). Which behaviour in convergence rates do you observe? Describe it qualitatively or create a table/plot as in lecture notes.

4. Change your grid to unstructured triangular grid and choose $k_1 = 1, k_2 = 10$. Why are convergence rates not as expected?

6 points