

Shared Memory Programming Models II

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WS 14/15

Parallel Programming Models II

Communication using shared memory

- Barrier – synchronization of all processes
- Semaphores
- Philosophers problem

Global Synchronization

- *Barrier*: All processors shall wait on each other until all have arrived
- Barriers are often repeatedly executed repeatedly:

```
while (1) {  
    a calculation;  
    Barrier;  
}
```

- Since the calculation is load balanced, all arrive simultaneously at the barrier
- First idea: Count all arriving processes

Global Synchronization

Program (First proposal of a barrier)

parallel *barrier-1*

```
{
  const int P=8;      int count=0;      int release=0;

  process  $\Pi$  [int  $p \in \{0, \dots, P - 1\}$ ]
  {
    while (1)
    {
      calculation;
      CSenter;           // entry
      if (count==0) release=0;      // reset
      count=count+1;      // increment counter
      CSexit;           // exit
      if (count==P) {
        count=0;           // last resets counter
        release=1;       // and frees
      }
      else while (release==0) ;      // waiting
    }
  }
}
```

Barrier with Sense Reversal

Wait *reversible* for *release*==1 and *release*==0

Program (Barrier with direction reversal)

parallel *sense-reversing-barrier*

```
{
  const int P=8;    int count=0;    int release=0;

  process  $\Pi$  [int  $p \in \{0, \dots, P - 1\}$ ]
  {
    int local_sense = release;
    while (1)
    {
      calculation;
      local_sense = 1-local_sense;           // change direction
      CSenter;                               // entry
      count=count+1;                         // increment counter
      CSexit;                                // exit
      if (count==P) {
        count=0;                             // last resets
        release=local_sense;                 // and frees
      } else
        while (release!=local_sense) ;
    }
  }
}
```

Complexity is $O(P^2)$ since all P processes have to pass through a critical section at a time. Is there a better approach?

Hierarchical Barrier: Variant 1

In the barrier with counter all P processes have to pass through a critical section. This necessitates $O(P^2)$ memory accesses. We now develop a solution with $O(P \log P)$ accesses.

We start with *two* processes and consider the following program segment:

```
int arrived=0, continue=0;
```

```
 $\Pi_0$ :
```

```
while ( $\neg$ arrived) ;  
arrived=0;  
continue=1;
```

```
 $\Pi_1$ :
```

```
arrived=1;
```

```
while ( $\neg$ continue) ;  
continue=0;
```

We use two synchronization variables, so called *flags*

Hierarchical Barrier: Variant 1

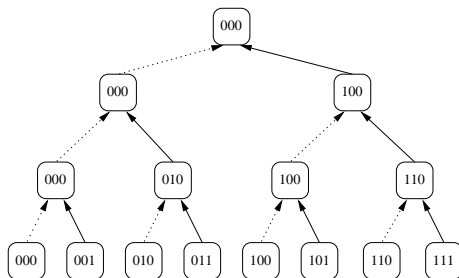
When using flags the following rules have to be met:

- 1 The process, that waits for a flag, also resets it.
- 2 A flag may first be newly set, if it has been safely reset.

Both rules are respected by our solution.

The solution assumes sequential consistency of the memory!

We now apply this idea in a hierarchical way:



Hierarchical Barrier: Variant 1

Program (Barrier with tree)

parallel *tree-barrier*

```
{
  const int  $d = 4$ ,  $P = 2^d$ ; int arrived[ $P$ ]={0[ $P$ ]}, continue[ $P$ ]={0[ $P$ ]};

  process  $\Pi$  [int  $p \in \{0, \dots, P - 1\}$ ]
  {
    int  $i, r, m, k$ ;
    while (1) {
      calculation;
      for ( $i = 0; i < d; i++$ ) { // upward
         $r = p \& \left[ \sim \left( \sum_{k=0}^i 2^k \right) \right]$ ; // reset bits 0 to i
         $m = r \mid 2^i$ ; // set bit i
        if ( $p == m$ ) arrived[ $m$ ]=1;
        if ( $p == r$ ) {
          while ( $\neg$ arrived[ $m$ ]) ; // wait
          arrived[ $m$ ]=0;
        }
      }
    } // process 0 knows that all are there
  }
  ...
}
```


Hierarchical Barrier: Variant 1

Program (Barrier with tree cont.)

parallel tree-barrier cont.

```
{  
  
    ...  
    for ( $i = d - 1; i \geq 0; i --$ ) {           // downward  
         $r = p \& \left[ \sim \left( \sum_{k=0}^i 2^k \right) \right];$  // reset bits 0 to i  
         $m = r | 2^i;$   
        if ( $p == m$ ) {  
            while( $\neg \text{continue}[m]$ ) ;  
             $\text{continue}[m]=0;$   
        }  
        if ( $p == r$ )  $\text{continue}[m]=1;$   
    }  
}  
}
```

Caution: Flag variables should be stored in different cache lines, that accesses do not hinder themselves!

Hierarchical Barrier: Variant 2

This variant presents a symmetric solution of the barrier with *recursive doubling*.

We consider at first again the barrier for two processes Π_i and Π_j :

Π_i :

```
while (arrived[i]) ;  
arrived[i]=1;  
while ( $\neg$ arrived[j]) ;  
arrived[j]=0;
```

Π_j :

```
while (arrived[j]) ;  
arrived[j]=1;  
while ( $\neg$ arrived[i]) ;  
arrived[i]=0;
```

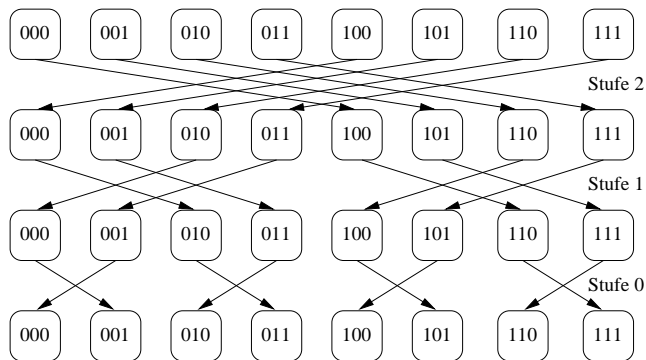
As prerequisite for the general solution the flags are organized as arrays, in the beginning all flags are 0.

Sequence in words:

- Line 2: Each sets *its* flag to 1
- Line 3: Each waits onto the flag of the other
- Line 4: Each resets the flag of the *other*
- Line 1: Because of rule 2 from above wait until the flag is reset
- Now we use this ideas in a recursive manner!

Hierarchical Barrier: Variant 2

Recursive doubling uses the following communication structure:



- No idle processors
- Each step is a two way communication

Hierarchical Barrier: Variant 2

Program (Barrier with recursive doubling)

parallel *recursive-doubling-barrier*

```
{
  const int  $d = 4$ ,       $P = 2^d$ ; int arrived[ $d$ ][ $P$ ]= $\{0[P \cdot d]\}$ ;

  process  $\Pi$  [int  $p \in \{0, \dots, P - 1\}$ ]
  {
    int  $i, q$ ;
    while (1) {
      calculation;
      for ( $i = 0; i < d; i++$ )                                // all steps
      {
         $q = p \oplus 2^i$ ;                                       // reverse bit  $i$ 
        while (arrived[ $i$ ][ $p$ ]) ;
        arrived[ $i$ ][ $p$ ]=1;
        while ( $\neg$ arrived[ $i$ ][ $q$ ]) ;
        arrived[ $i$ ][ $q$ ]=0;
      }
    }
  }
}
```

Semaphore

A semaphore is an abstraction of a synchronisation variable, that enables the elegant solution of multiple synchronisation problems

Up-to-now all programs have used *active waiting*. This is very inefficient under quasi-parallel processing of multiple processes on one processor (multitasking). The semaphore enables to switch processes into an idle state.

We understand a semaphore as abstract data type: Data structure with operations, that fulfill particular properties:

A semaphore S has a non-negative integer value $value(S)$, that is assigned during creation of the semaphore with the value *init*.

For a semaphore S two operations $\mathbf{P}(S)$ and $\mathbf{V}(S)$ are defined with:

- $\mathbf{P}(S)$ decrements the value of S by one if $value(S) > 0$, otherwise the process *blocks* as long as another process executes a \mathbf{V} operation on S .
- $\mathbf{V}(S)$ frees another process from a \mathbf{P} operation if one is waiting (are several waiting one is selected), otherwise the value of S is incremented by one. \mathbf{V} operations never block!

Semaphore

Is the number of *successfully finished* **P** operations n_P and the one of **V** operations n_V , then for the value of the semaphore applies always:

$$\text{value}(S) = n_V + \text{init} - n_P \geq 0$$

or equivalent $n_P \leq n_V + \text{init}$.

The value of a semaphore is *not* visible from the outside. It shows only by the executability of the **P** operation.

The increment resp. decrement of a semaphore is performed in an atomic way, multiple processes can also perform **P/V** operations concurrently.

Semaphores, that can take a value larger than one, are called *general semaphores*.

Semaphores, that only have values $\{0, 1\}$, are called *binary semaphores*.

Notation:

Semaphore $S=1$;

Semaphore $\text{forks}[5] = \{1 [5]\}$;

Mutual Exclusion with Semaphore

We now present in which way all already treated synchronisation problems can be solved with semaphore variables. The first application is dedicated to mutual exclusion by usage of a single binary semaphore:

Program (Mutual exclusion with semaphore)

```
parallel cs-semaphore
{
  const int P=8;
  Semaphore mutex=1;
  process  $\Pi$  [int i  $\in \{0, \dots, P - 1\}$ ]
  {
    while (1)
    {
      P(mutex);
      critical section;
      V(mutex);
      uncritical section;
    }
  }
}
```

Mutual Exclusion with Semaphore

By multitasking processes can be switched to the idle state (waiting).

Fairness is easy to integrate into the wake-up mechanism (FCFS).

Memory consistency model can be respected by the implementation, programs remains portable (e. g. Pthreads)

Barrier with Semaphore

- Each process has to be delayed until the other(s) arrive at the barrier.
- The barrier has to be reusable, since it is usually executed several times.

Program (Barrier with semaphore for two processes)

```
parallel barrier-2-semaphore
```

```
{  
  Semaphore b1=0, b2=0;  
  process  $\Pi_1$   
  {  
    while (1) {  
      calculation;  
      V(b1);  
      P(b2);  
    }  
  }  
}
```

```
process  $\Pi_2$   
{  
  while (1) {  
    calculation;  
    V(b2);  
    P(b1);  
  }  
}
```

Barrier with Semaphore

After unrolling of the loop, the code looks as follows:

Π_1 :	Π_2 :
calculation 1;	calculation 1;
$V(b1)$;	$V(b2)$;
$P(b2)$;	$P(b1)$;
calculation 2;	calculation 2;
$V(b1)$;	$V(b2)$;
$P(b2)$;	$P(b1)$;
calculation 3;	calculation 3;
$V(b1)$;	$V(b2)$;
$P(b2)$;	$P(b1)$;
...	...

Assume process Π_1 works in calculation phase i , thus it has executed $P(b2)$ $i - 1$ -times. Assume further Π_2 works in calculation phase $j < i$, therefore it has executed $V(b2)$ $j - 1$. Then holds

$$n_P(b2) = i - 1 > j - 1 = n_V(b2).$$

On the other hand the semaphore rules assure, that

$$n_P(b2) \leq n_V(b2) + 0.$$

This is a contradiction and it can not apply $j < i$. The argument is symmetric and applies also when the processor numbers are exchanged.

Producer/Consumer $m/n/1$

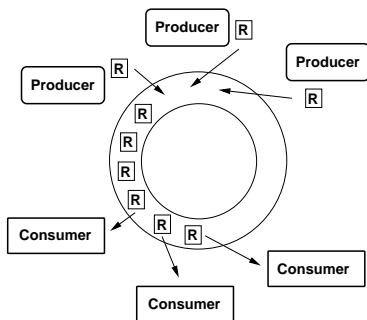
m producers, n consumers, 1 buffer location,

Producer has to block if the buffer location is occupied.

Consumer has to block if no request is stored.

We use two semaphores:

- *empty*: counts number of *free* buffer locations
- *full*: counts number of *occupied* locations (requests)



Produce/Consumer $m/n/1$

Program (m producer, n consumer, 1 buffer location)

```
parallel prod-con-nm1
```

```
{  
    const int m = 3, n = 5;  
    Semaphore empty=1;           // free buffer location  
    Semaphore full=0;           // available request  
    T buf;                       // the buffer  
    process P [int i ∈ {0, ..., m - 1}] {  
        while (1) {  
            Generate request t;  
            P(empty);           // Is buffer free?  
            buf = t;           // store request  
            V(full);           // request available  
        }  
    }  
    process C [int j ∈ {0, ..., n - 1}] {  
        while (1) {  
            P(full);           // Is request available?  
            t = buf;           // remove request  
            V(empty);         // buffer is empty  
            Process request t;  
        }  
    }  
}
```

Shared binary semaphore (*split binary semaphore*):

$$0 \leq \text{empty} + \text{full} \leq 1 \quad (\text{invariant})$$

Producer/Consumer 1/1/k

1 producer, 1 consumer, k buffer locations,

Buffer is array of length k of type T . Insertion and deletion works with

$$buf[front] = t; \quad front = (front + 1) \bmod k;$$
$$t = buf[rear]; \quad rear = (rear + 1) \bmod k;$$

Semaphore as above, only initialized with k !

Program (1 producer, 1 consumer, k buffer locations)

```
parallel prod-con-11k
```

```
{
```

```
    const int k = 20;
```

```
    Semaphore empty=k;
```

```
    // counts free buffer locs
```

```
    Semaphore full=0;
```

```
    // count available requests
```

```
    T buf[k];
```

```
    // the buffer
```

```
    int front=0;
```

```
    // newest request
```

```
    int rear=0;
```

```
    // oldest request
```

```
}
```

Producer/Consumer 1/1/k

Program (1 producer, 1 consumer, k buffer locations)

parallel *prod-con-11k*

```
{  
  process P {  
    while (1) {  
      Generate request t;  
      P(empty);           // Is buffer free?  
      buf[front] = t;    // store request  
      front = (front+1) mod k; // next free location  
      V(full);         // request available  
    }  
  }  
  
  process C {  
    while (1) {  
      P(full);         // Is request there?  
      t = buf[rear];    // remove request  
      rear = (rear+1) mod k; // next request  
      V(empty);      // buffer is free  
      Process request t;  
    }  
  }  
}
```

Producer/Consumer $m/n/k$

m producers, n consumers, k buffer locations,

We only have to ensure, that producers among each other and consumers cannot manipulate the buffer at the same time.

Use two additional binary semaphores $mutexP$ und $mutexC$

Program (m producer, n consumer, k buffer locations)

```
parallel prod-con-mnk
```

```
{  
    const int k = 20, m = 3, n = 6;  
    Semaphore empty=k;           // count free buffer locations  
    Semaphore full=0;            // count available requests  
    T buf[k];                    // the buffer  
    int front=0;                 // newest request  
    int rear=0;                  // oldest request  
    Semaphore mutexP=1;         // access of producers  
    Semaphore mutexC=1;         // access of consumersr  
}
```

Producer/Consumer $m/n/k$

Program (m producer, n consumer, k buffer locations)

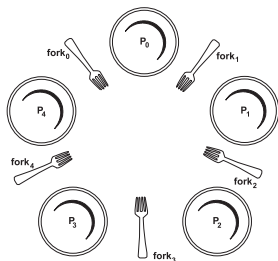
parallel process

```
{
  P [int  $i \in \{0, \dots, m - 1\}$ ] {
    while (1) {
      Generate request  $t$ ;
      P(empty); // Is buffer free?
      P(mutexP); // manipulate buffer
      buf[front] =  $t$ ; // store request
      front = (front+1) mod  $k$ ; // next free position
      V(mutexP); // ready with buffer
      V(full); // request available
    }
  }
  process C [int  $j \in \{0, \dots, n - 1\}$ ] {
    while (1) {
      P(full); // Is request there?
      P(mutexC); // manipulate buffer
       $t = \text{buf}[\text{rear}]$ ; // remove request
      rear = (rear+1) mod  $k$ ; // next request
      V(mutexC); // ready with buffer
      V(empty); // buffer is free
      Process request  $t$ ;
    }
  }
}
```


Dining Philosophers

Complex synchronisation task: A process necessitates exclusive access onto several resources to perform a specific task.

→ overlapping critical sections.



Five philosophers sit at a round table. The exercise of each philosopher consists out of interchanging phases of thinking and eating. In between two of the philosophers a fork is positioned and in the center of the table a mountain Spaghetti is located. To eat a philosopher needs two forks – the one laying left and right next to him.

Dining Philosophers

The problem:

Write a parallel program, with one process per philosopher, that

- enables a maximal count of philosophers to eat and
- that avoids a deadlock.

Skeletal structure of a philosopher:

```
while (1)
{
    think;
    take forks;
    eat;
    lay back forks;
}
```

Naive Philosophers

Program (Naive solution of the philosophers problem)

```
parallel philosophers-1
```

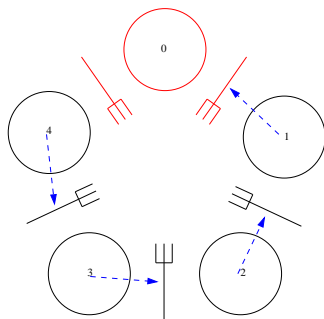
```
{  
  const int P = 5;                                // number of philosophers  
  Semaphore forks[P] = { 1 [P] };             // forks  
  
  process Philosopher [int p ∈ {0, ..., P - 1}] {  
    while (1) {  
      Thinking;  
      P(fork[p]);                                // left fork  
      P(fork[(p + 1) mod P]);                   // right fork  
      Eating;  
      V(fork[p]);                                // left fork  
      V(fork[(p + 1) mod P]);                   // right fork  
    }  
  }  
}
```

Naive Philosophers

Philosophers are deadlocked, if all take at first the left fork!

Simple solution of the deadlock problem: Avoid cyclic dependencies, e. g. philosopher 0 takes his forks in a different sequence right then left.

This solution allows eventually not maximal concurrency:



Clever Philosophers

Take forks only, when *both* are available

Critical section: only one can manipulate the forks

Three states of a philosopher: thinking, hungry, eating

Program (Solution of philosophers problem)

```
parallel philosophers-2
{
    const int P = 5;                               // count philosophers
    const int think=0, hungry=1, eat=2;
    Semaphore mutex=1;
    Semaphore s[P] = { 0 [P] };                   // eating philosopher
    int state[P] = { think [P] };                  // state
}
```

Clever Philosophers

Program (Solution of philosophers problem)

parallel process

```
{
  Philosopher [int p ∈ {0, ..., P - 1}] {
    void test (int i) {
      int l=(i + P - 1) mod P, r=(i + 1) mod P;
      if (state[i]==hungry ∧ state[l]≠eat ∧ state[r]≠eat)
      {
        state[i] = eat;
        V(s[i]);
      }
    }

    while (1) {
      Thinking;
      P(mutex); // take forks
      state[p] = hungry;
      test(p);
      V(mutex);
      P(s[p]); // wait, if neighbor eats
      Eating;
      P(mutex); // lay forks downs
      state[p] = think;
      test((p + P - 1) mod P); // wake-up left neighbor
      test((p + 1) mod P); // wake-up right neighbor
      V(mutex);
    }
  }
}
```